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1 Turing Machine

1.1 DFA and PDA and TM

(a) DFA

- DFA contains finite states.
- When staying in the current state, read a symbol from input string, then move to another state according to transition function $\sigma\colon Q \times \Sigma \to Q$.
- Accept input string when reach an accept state after read whole input string.

(b) PDA

- PDA contains finite states and a infinite stack (memory).
- When staying in the current state, read a symbol from input string and pop a symbol from the stack, then push a symbol to the stack and move to another state according to transition function $\sigma\colon Q \times \Sigma \times \Gamma \to 2^{Q \times \Gamma}$.
- Accept input string when we reach an accept state after read whole input string.

(c) TM

- TM contains finite states and a infinite tape(input string + memory).
- When staying in the current state, read a symbol from the tape, then write a symbol to the tape, move to another state and move the head to the left or the right according to transition function $\sigma\colon Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$.
- Halt immediately when reach accept state or reject state.

1.2 Formal description of TM

A Turning machine is a $7-$tuple, $(Q, \Sigma, \Gamma, \sigma, q_0, q_{accept}, q_{reject})$, where Q, Σ, Γ are all finite sets.

(a) Q is the set of states,

(b) Σ is the input alphabet not containing the blank symbol $\sqcup$,

(c) Γ is the tape alphabet, where $\sqcup \in \Gamma$ and $\Sigma \subset \Gamma$.

(d) $\sigma\colon Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$. is the transition function.

(e) $q_0 \in Q$ is the start state.

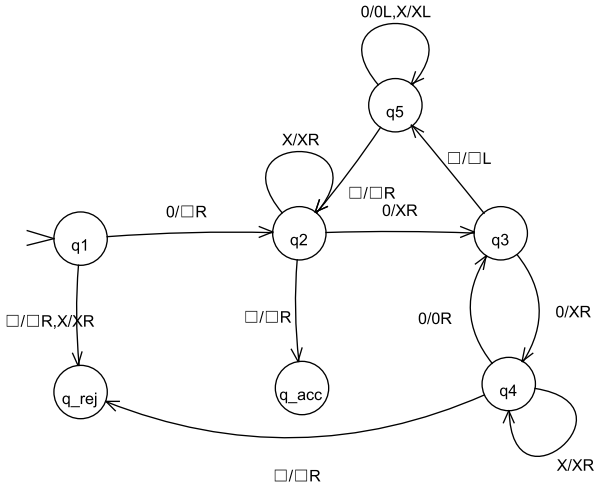
(f) $q_{accept} \in Q$ is the accept state, and

(g) $q_{reject} \in Q$ is the reject state, where $q_{reject} \neq q_{accept}$.

2 An Example

Describe a Turning machine(TM) M_2 that decides $A = \{0^{2^n} | n \geq 0\}$, the language consisting of all string of $0s$ whose length is a power of 2.

- A high level description of $M_2 =$ " On input string w:
  (a) Sweep left to right across the tape, crossing off every other 0.
  (b) If in stage (a) the tape contained a single 0, accept.
  (c) If in stage (a) the tape contained more than a single and the number of $0s$ was odd, reject.
  (d) Return the head to the left-hand end of the tape. Go to stage (a)."
- State diagram for Turing machine M_2.



Here $\Gamma = \{0, x, \square\}$. We can cross off every other 0 in part $q3, q4$. If the tape contains just a single 0 we will accept the string in part $q1, q2, q\_acc$. And if the tape contains more than a single 0 and the number of $0s$ is odd we will reject the string in part $q4, q\_rej$.