CSI 2101 B/C: Discrete Structures — Homework 4

University of Ottawa, Winter 2024

1. In which of the following Die Hard scenarios does Bruce survive? Justify your answer.
   (a) Target 5ℓ, jug capacities 7ℓ and 4ℓ.
   (b) Target 12ℓ, jug capacities 182ℓ and 217ℓ.
   (c) Target $\frac{1}{2}\ell$, jug capacities $6\frac{1}{4}\ell$ and $11\frac{1}{4}\ell$.
   (d) **(Optional)** Target 6ℓ, jug capacities 16ℓ, 28ℓ, and 36ℓ.

2. Apply the extended GCD algorithm to find a representation of $\gcd(a, b)$ as a combination $sa + tb$ of a and b given below. The two coefficients s and t will have different signs. Then find another combination with the signs reversed.
   (a) $a = 105$ and $b = 42$
   (b) $a = 2002$ and $b = 1881$

3. Here is another algorithm G for calculating GCDs. It assumes the inputs a and b are positive integers.

   $G(a, b)$:
   if $a = b$, output a.
   if $a > b$, output $G(a - b, b)$
   otherwise, output $G(a, b - a)$.

   (a) Viewing G as a state machine, show the states that the algorithm visits on inputs $a = 27$ and $b = 6$.
   (b) Prove that the GCD of the two arguments stays the same throughout the execution.
   (c) Use part (b) to prove that $G(a, b)$ outputs the GCD of a and b assuming that it has terminated.
   (d) Prove that G always terminates (**Hint:** There is a quantity that decreases in every step.)

4. For each of the following statements about integers, say if it is true or false. Justify your claim with a proof.
   (a) If c divides $a + b$ then c divides a and c divides b.
   (b) If $\gcd(a, c) = 1$ and $\gcd(b, c) = 1$ then $\gcd(ab, c) = 1$.
   (**Hint:** Use the connection between gcd and combinations.)
   (c) For all $n \geq 1$, $\gcd(21n + 4, 14n + 3) = 1$.