1. Calculate the following numbers.

   (a) $98 + 96 + 94 + 92 + 90 \bmod 100$

   (b) $17 \cdot 23 - 2 \cdot 3 \bmod 17$

   (c) $9^{-1} \bmod 23$

   (d) $95 \cdot 41^{-1} \bmod 97$. (97 is a prime number.)

2. Calculate the following numbers using the suggested method:

   (a) $2^9 \bmod 11$ using iterated multiplication.

   (b) $2^{81} \bmod 11$ using fast exponentiation (the *Power* algorithm from Lecture 5).

   (c) $2^{2^{81}} \bmod 11$ using Fermat's Little Theorem (Theorem 5 from Lecture 5).

3. Calculate the following numbers.

   (a) x and y that solve $5x + 7y \equiv 17 \pmod{19}$ and $4x + 11y \equiv 13 \pmod{19}$.

   (b) $1^1 + 2^2 + \cdots + 99^{99} \bmod 3$.

   (c) $1^{-1} + 2^{-1} + \cdots + 96^{-1} \bmod 97$.

   (d) **(Optional)** $42! \bmod 43$ (*Hint:* Pair up each number with its inverse. You can try $6! \bmod 7$ first.)

4. You will investigate the "baby RSA" encryption from Lecture 5. Recall that the public encryption key e and "secret" decryption key d are chosen so that $ed \equiv 1 \pmod{n-1}$ for prime modulus n.

   (a) Assume $n = 29$ and $d = 11$. Show how to choose e to enable decryption.

   (b) Calculate the encryption $c = m^e \bmod n$ of the message $m = 10$ and the encryption key e from part (a). Then calculate the decryption $c^d \bmod n$.

   (c) Now suppose Eve observes the ciphertext $c = 33$ that Alice sent to Bob using modulus $n = 37$ and encryption key $e = 7$. How can Eve recover the message m without knowing d?