1. Find exact closed form expressions for the following sums. Explain how you discovered the expression and prove that it is correct.

   (a) $1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2$.

   (b) $3^n + 3^{n+1} + 3^{n+2} + \cdots + 3^{2n}$.

   (c) (**Optional**) $1/2 + 2/2^2 + 3/2^3 + \cdots + n/2^n$.
   (**Hint:** Call this number S. Subtract S from $2S$ term by term.)

2. Show the following inequalities by using the integral method for approximating sums.

   (a) $2\sqrt{n+1} - 2 \leq 1/\sqrt{1} + 1/\sqrt{2} + \cdots + 1/\sqrt{n} \leq 2\sqrt{n+1} - 1$.

   (b) $n^3/3 \leq 1^2 + 2^2 + \cdots + n^2 \leq n^3/3 + n^2$.

   (c) $1 \cdot e^{-1^2} + 2 \cdot e^{-2^2} + \cdots + n \cdot e^{-n^2} \leq 3/(2e)$.

3. Sort the following functions in increasing order of asymptotic growth:

$$2^n, n^n, e^{2^n}, 2^{e^n}, n^{e^2}.$$

   (For example, if you are given the functions n^2, n, and 2^n, the sorted list would be $n, n^2, 2^n$.) Show that for every pair of consecutive functions f, g in your list, f is $o(g)$.

4. Write each of the following summations S as big-theta of a simple closed-form function f. Prove that S is $O(f)$ and f is $O(S)$.

   (a) $n + (n+1) + (n+2) + \cdots + 2n$.

   (b) $\log(n) + \log(n+1) + \cdots + \log(2n)$.

   (c) $2^{1^2} + 2^{2^2} + \cdots + 2^{n^2}$. (**Hint:** Use the geometric sum formula.)