Homework 8

1. For each of the following functions, say if it is (i) injective (ii) surjective. Justify your answer.
   (a) $f\colon \{0,1\}^3 \to \{0,1,2,3\}$ given by $f((x,y,z)) = x + y + z$.
   (b) $g\colon \{0,1\}^3 \to \{0,1,2,3,4,5,6,7\}$ given by $g((x,y,z)) = x + 2y + 4z$.
   (c) $h\colon \{0,1\}^3 \to \{0,1,3,4,5,6,7,8\}$ given by $h((x,y,z)) = x + 3y + 4z$.
2. A password consists of the digits `0` to `9` and the special symbols `*` and `#`. How many 6 to 8-symbol passwords are there if
   (a) the password starts with a `*` and ends with a `#`?
   (b) there is at least one special symbol?
   (c) there is exactly one `*` and exactly two `#`s?
3. How many 8×8 chessboard configurations are there with...
   (a) 4 white rooks, and all must be in different rows and columns?
   (b) 2 white and 2 black rooks, and all must be in different rows and columns?
   (c) 2 white and 2 black rooks, and all rooks of the same color must be in different rows and columns? (**Hint:** Apply the sum rule after fixing the white rooks' positions.)
4. Use the pigeonhole principle to prove that
   (a) Among any 17 points in the unit square there is a pair within distance at most 0.36.
   (b) In every set of 14 numbers between 0 and 42 there are three pairs that have the same sum modulo 43.
   (c) In every group of at least two people there are two that have the same number of friends within the group. (**Hint:** Assume first that everyone has at least one friend.)