1. Consider the following graph properties and determine if such graphs exist. If they do, provide an example. If not, provide a proof of their non-existence.

   (a) A graph with 100 vertices of degree 3 and 3 vertices of degree 99.

   (b) A graph with 100 vertices of degree 2 and 2 vertices of degree 99.

   (c) A bipartite graph with 100 vertices of degree 2 and 2 vertices of degree 99.

2. A science fair has participants from schools in X, Y, and Z cities. The table entry $P(r, c)$ in row r and column c represents the average number of projects completed by students from city r in collaboration with students from city c:

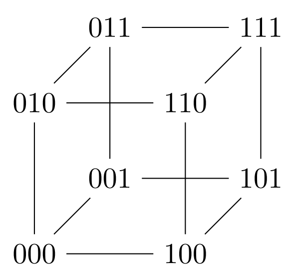
	X	Y	Z
X	4	?	2
Y	3	5	1
Z	6	2	3

   (a) Show that $P(r, c)/P(c, r)$ must equal (number of students from c)/(number of students from r).

   (b) Use part (a) to show that $P(X, Y) \cdot P(Y, Z) \cdot P(Z, X) = P(Y, X) \cdot P(Z, Y) \cdot P(X, Z)$.

   (c) Find the missing entry in the table.

3. The *n-dimensional cube* Q_n is a graph on 2^n vertices in which the vertices are all bit strings of length n. Two vertices are adjacent if they differ in exactly one position. Here is a diagram of Q_3:



   (a) Show that for every $n \geq 1$, Q_n is a bipartite graph.

   (b) Show that for every $n \geq 1$, Q_n has a perfect matching.

   (c) Assuming n is odd, let R_n be the graph obtained by removing all vertices from Q_n except those that have exactly $(n - 1)/2$ zeroes or ones. Show that R_n is (i) bipartite and (ii) regular.

   (d) By part (d) R_n has a perfect matching for all odd n. Describe perfect matchings for R_3 and R_5.

4. Find stable matchings for the following preference lists with (a) boys proposing and girls choosing and (b) girls proposing and boys choosing.