

Prove that $(\sqrt{2} - 1)^2$ is Irrational. Specify your proof method.

Solution: We prove this by contradiction. Suppose $(\sqrt{2} - 1)^2$ is a rational number r . Then

$$r^2 = (\sqrt{2} - 1)^2 = 2 - 2\sqrt{2} + 1 = 3 - 2\sqrt{2}.$$

Therefore $\sqrt{2} = (3 - r^2)/2$ which is a rational number. This contradicts the theorem we proved in Lecture 2.