

Which of the following is an integer linear combination of 28 and 76?

1 6 -16 28

Circle all such numbers (if any). Justify your claim. Explain all calculations.

Solution: By running Euclid's algorithm we find that

$$\begin{aligned} E(76, 28) &= E(28, 20) & 76 &= 2 \cdot 28 + 20 \\ &= E(20, 8) & 28 &= 1 \cdot 20 + 8 \\ &= E(8, 4) & 20 &= 2 \cdot 8 + 4 \\ &= E(4, 0) & 8 &= 2 \cdot 4 + 0 \end{aligned}$$

the GCD of 76 and 28 is 4. (Alternatively the GCD can be recovered from the prime factorizations $28 = 2^2 \cdot 7$, $76 = 2^2 \cdot 19$.) The linear combinations are the multiples of the GCD, which rules out 1 and 6 and leaves in -16 and 28.