

1. For each of the following functions, say if it is (i) injective (ii) surjective. Justify your answer.
  - (a)  $f: \{0, 1\}^3 \rightarrow \{0, 1, 2, 3\}$  given by  $f((x, y, z)) = x + y + z$ .
  - (b)  $g: \{0, 1\}^3 \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7\}$  given by  $g((x, y, z)) = x + 2y + 4z$ .
  - (c)  $h: \{0, 1\}^3 \rightarrow \{0, 1, 3, 4, 5, 6, 7, 8\}$  given by  $h((x, y, z)) = x + 3y + 4z$ .
2. A password consists of the digits 0 to 9 and the special symbols \* and #. How many 6 to 8-symbol passwords are there if
  - (a) the password starts with a \* and ends with a #?
  - (b) there is at least one special symbol?
  - (c) there is exactly one \* and exactly two #s?
3. How many  $8 \times 8$  chessboard configurations are there with...
  - (a) 4 white rooks, and all must be in different rows and columns?
  - (b) 2 white and 2 black rooks, and all must be in different rows and columns?
  - (c) 3 white rooks and 1 black rook, and all white rooks must be in different rows and columns?
  - (d) (**Optional**) 2 white and 2 black rooks, and all rooks of the same color must be in different rows and columns? (**Hint:** Apply the sum rule after fixing the white rooks' positions.)
4. Use the pigeonhole principle to prove that
  - (a) Among any 17 points in the unit square there is a pair within distance at most 0.36.
  - (b) In every set of 14 numbers between 0 and 42 there are three pairs that have the same sum modulo 43.
  - (c) In every group of at least two people there are two that have the same number of friends within the group. (**Hint:** Assume first that everyone has at least one friend.)