1. Consider the following graph properties and determine if such graphs exist. If they do, provide an example. If not, provide a proof of their non-existence.

   (a) A graph with 100 vertices of degree 3 and 3 vertices of degree 99.

   (b) A graph with 100 vertices of degree 2 and 2 vertices of degree 99.

   (c) A bipartite graph with 100 vertices of degree 2 and 2 vertices of degree 99.

2. A summer camp has students from *A*tlanta, *B*oston, and *C*hicago. The table entry $P(r, c)$ in row r and column c represents the average number of friends students from city r made with students from city c:

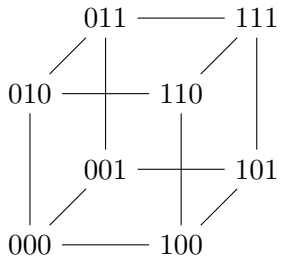
	A	B	C
A	4	?	2
B	3	5	1
C	6	2	3

   (a) Show that $P(r, c)/P(c, r)$ must equal (number of students from c)/(number of students from r).

   (b) Use part (a) to show that $P(A, B) \cdot P(B, C) \cdot P(C, A) = P(B, A) \cdot P(C, B) \cdot P(A, C)$.

   (c) Find the missing entry in the table.

3. The *n-dimensional cube* Q_n is a graph on 2^n vertices in which the vertices are all bit strings of length n. Two vertices are adjacent if they differ in exactly one position. Here is a diagram of Q_3:



   (a) Show that for every $n \geq 1$, Q_n is a bipartite graph.

   (b) Show that for every $n \geq 1$, Q_n has a perfect matching.

   (c) Assuming n is odd, let R_n be the graph obtained by removing all vertices from Q_n except those that have exactly $(n-1)/2$ zeroes or ones. Show that R_n is (i) bipartite and (ii) regular.

   (d) By part (d) R_n has a perfect matching for all odd n. Describe perfect matchings for R_3 and R_5.

4. Find stable matchings for the following preference lists using the Gale-Shapley algorithm with (a) boys proposing and girls choosing and (b) girls proposing and boys choosing.