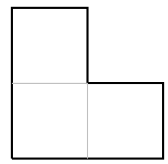
Underline and explain the mistake in the following "proof".

Proposition: For every $n \geq 1$, the 2^n by 2^n square grid with any *four* squares removed can be completely tiled using the following type of tile:



Proof. We prove the proposition by induction on n.

Base case $n = 1$: The 2×2 grid with four squares removed has no squares remaining, so no tiles are needed.

Inductive step: Assume every 2^n by 2^n square grid with any four squares removed can be completely tiled. Let G be a 2^{n+1} by 2^{n+1} grid. Divide G into four 2^n by 2^n quadrants. <u>By the inductive hypothesis, all four quadrants can be tiled.</u> Therefore G can also be tiled. □

Solution: The inductive hypothesis only guarantees that each quadrant can be tiled provided that it has four squares removed. The four quadrants of G, however, will not have four squares removed. Therefore the inductive hypothesis cannot be applied.

For example, the instance below (with $n = 2$) has four squares removed, but each of the quadrants has zero or two squares removed. The inductive hypothesis cannot be applied to any of them.