

Can you measure 10 litres of water with a 198 litre jug and a 168 litre jug?
Justify your answer. Show your calculations.

Solution: No. On inputs 198 and 168, Euclid's algorithm

$$\begin{aligned} E(198, 168) &= E(168, 30) & 198 &= 1 \cdot 168 + 30 \\ &= E(30, 18) & 168 &= 5 \cdot 30 + 18 \\ &= E(18, 12) & 18 &= 1 \cdot 12 + 6 \\ &= E(12, 6) & 12 &= 2 \cdot 6 + 0 \\ &= E(6, 0) \\ &= 6 \end{aligned}$$

outputs 6 for their GCD. As 6 divides any combination of 198 and 168 it must divide all measurable amounts. But 6 does not divide 10 so 10 litres cannot be measured.