

Let

$$f(n) = 1^n + 2^n + 3^n \quad g(n) = n + n^2 + n^3 \quad h(n) = n^2 \cdot 2^n.$$

Circle one alternative in each row. Justify your answer.

f is $o(g)$ or g is $o(f)$

g is $o(h)$ or h is $o(g)$

h is $o(f)$ or f is $o(h)$

Solution: We first identify f and g with their big-Theta equivalent: $f(n)$ is $\Theta(3^n)$ (as it is between 3^n and $3 \cdot 3^n$), and $g(n)$ is $\Theta(n^3)$ (as it is between n^3 and $3n^3$).

g is $o(h)$ as g is (Θ -equivalent to) a polynomial, and h is an exponential: $g(n)/h(n) = \Theta(n/2^n)$ goes to zero.

h is $o(f)$ as f is an exponential with a larger base: $h(n)/f(n) = \Theta(n^2/(3/2)^n)$ goes to zero.

Finally, g is $o(f)$ because g is $o(h)$ and h is $o(f)$.