Let $f(n)$ be the number of length-n sequences of `A`s and `B`s in which no two `A`s are consecutive. For example, when $n = 2$, the sequences `AB`, `BA`, and `BB` are counted but `AA` is not, so $f(2) = 3$.

Write a recurrence that expresses $f(n)$ in terms of $f(n-1)$ and $f(n-2)$. Explain how you obtained it. Use it to calculate $f(6)$.

Solution: If the first symbol is a `B`, the remaining $(n-1)$-symbol sequence can be arbitrary without two consecutive `A`s. There are $f(n-1)$ sequences of this type. If the first symbol is an `A`, it must be followed by a `B`, and then by an arbitrary length-$(n-2)$ sequence without two consecutive `A`s. Therefore

$$f(n) = f(n-1) + f(n-2).$$

The initial conditions are $f(0) = 1$ (the empty sequence) and $f(1) = 2$ (the 1-symbol sequences `A`, `B`). Iterating the recurrence we obtain $f(2) = 3, f(3) = 5, f(4) = 8, f(5) = 13$, and $f(6) = 21$.