A 6-symbol password starts with a letter followed by 5 letters or digits. It must contain at least one digit. How many possible passwords are there?

(There are 52 letters and 10 digits. Your answer should be in the form of an expression that can be typed into a calculator in a few seconds.)

Solution: Without the at least one digit restriction, the possible passwords form the product set $L \times (L \cup D)^5$, where L and D are the letters and digits, respectively. To account for the one digit restriction we exclude the set $L \times L^5$ of a letter followed by 5 digits. As $L \times (L \cup D)^5$ is a disjoint union of the passwords P and the exclusions $L \times L^5$, by the sum and product rules

$$|P| = |L \times (L \cup D)^5| - |L \times L^5| = |L|\big((|L| + |D|)^5 - |L|^5\big) = 52 \cdot (62^5 - 52^5).$$