1. Which of the following pairs of statements are logically equivalent?

   (a) $P \longrightarrow (\text{NOT } Q)$
   $Q \longrightarrow (\text{NOT } P)$

   (b) If it is raining then the ground is wet.
   If the ground is not wet then it isn't raining.

   (c) If it is raining then the ground is wet.
   It is not raining or the ground is wet.

   (d) Not all three balls are of the same colour. (Colours are black or white.)
   Among the three balls, there is exactly one white ball, or there is exactly one black ball.

2. The Australian Open is a tennis tournament that is played in elimination rounds. 64 entrants are matched up to play, then so are the 32 winners of those matches, and so on until a single winner remains. $B(x, y)$ means that player x beat player y and $W(x)$ means that player x won the tournalent. Translate the following propositions into plain English. Which are true?

   (a) $\exists x, y : B(x, y) \text{ AND } W(y)$

   (b) $\forall x \exists y : B(x, y) \text{ OR } B(y, x)$

   (c) $\exists x, y, z : B(x, y) \text{ AND } B(y, z) \text{ AND } B(z, x)$

   (d) $\forall x : (\exists y : B(y, x)) \text{ IFF } (\text{NOT } W(x))$

   (e) $\exists x : \forall y : B(x, y) \longrightarrow \exists z : B(y, z)$

3. Express the following propositions about people and their relative heights using quantifiers and logical operators. Use x, y, z as variables and $T(x, y)$ for "x is taller than y". Make sure that all your variables are quantified. Explain your answer.

   (a) Bob is neither the tallest person nor the shortest person.

   (b) A person cannot be both taller and shorter than another person.

   (c) There is a shortest person, but there is no tallest person.

   (d) There are at least two people that are taller than Bob.
   (You can use $x \neq y$ for "x and y are different people".)

4. The following propositions are about students and the exams they pass: $E(s, c)$ means "Student s passed the exam in course c", and $P(s)$ means "Student s won a prize".

   (a) Explain the meaning of these two propositions in plain English:
   $A\colon \forall c\ \exists s\colon E(s, c) \text{ AND } (\text{NOT } P(s))$
   $B\colon \exists c\ \forall s\colon E(s, c) \longrightarrow P(s)$

   (b) Can A and B both be true? Justify your answer.

   (c) Explain the meaning of these two propositions in plain English:
   $C\colon \forall c\ \exists s\ \forall d\colon E(s, c) \text{ AND } ((c \neq d) \longrightarrow (\text{NOT } E(s, d)))$
   $D\colon \forall c, d\ \exists s\colon E(s, c) \text{ AND } ((c \neq d) \longrightarrow (\text{NOT } E(s, d)))$

   (d) Are C and D logically equivalent? Justify your answer.