

1. In which of the following Die Hard scenarios does Bruce survive? Justify your answer.
 - (a) Target 14ℓ , jug capacities 35ℓ and 63ℓ .
 - (b) Target $\frac{1}{2}\ell$, jug capacities $8\frac{1}{4}\ell$ and $14\frac{1}{4}\ell$.
 - (c) **(Optional)** Target 7ℓ , jug capacities 12ℓ , 18ℓ , and 30ℓ .
2. Apply the extended GCD algorithm to find a representation of $\gcd(a, b)$ as a combination $sa + tb$ of a and b given below. The two coefficients s and t will have different signs. Then find another combination with the signs reversed.
 - (a) $a = 105$ and $b = 42$
 - (b) $a = 2002$ and $b = 1881$
3. Here is another algorithm G for calculating GCDs. It assumes the inputs a and b are positive numbers.

$G(a, b)$:
if $a = b$, output a .
if $a > b$, output $G(a - b, b)$
otherwise, output $G(a, b - a)$.

 - (a) Viewing G as a state machine, show the states that the algorithm visits on inputs $a = 27$ and $b = 6$.
 - (b) Prove that the GCD of the two arguments stays the same throughout the execution.
 - (c) Use part (b) to prove that $G(a, b)$ outputs the GCD of a and b assuming that it has terminated.
 - (d) Prove that G always terminates (**Hint:** There is a quantity that decreases in every step.)
4. For each of the following statements about integers, say if it is true or false. Justify your claim with a proof.
 - (a) If c divides $a + b$ then c divides a and c divides b .
 - (b) If $\gcd(a, c) = 1$ and $\gcd(b, c) = 1$ then $\gcd(ab, c) = 1$.
(**Hint:** Use the connection between gcd and combinations.)
 - (c) For all $n \geq 1$, $\gcd(21n + 4, 14n + 3) = 1$.