1. Find exact closed form expressions for the following sums. Explain how you discovered the expression and prove that it is correct.
   (a) $1^2 + 3^2 + 5^2 + \cdots + (2n+1)^2$.
   (b) $(2^0 + \cdots + 2^n) + (2^1 + \cdots + 2^{n+1}) + \cdots + (2^n + \cdots + 2^{2n})$.
2. Show the following inequalities by using the integral method for approximating sums.
   (a) $2\sqrt{n+1} - 2 \le 1/\sqrt{1} + 1/\sqrt{2} + \cdots + 1/\sqrt{n} \le 2\sqrt{n+1} - 1$.
   (b) $n^3/3 \le 1^2 + 2^2 + \cdots + n^2 \le n^3/3 + n^2$.
   (c) $1 \cdot e^{-1^2} + 2 \cdot e^{-2^2} + \cdots + n \cdot e^{-n^2} \le 3/(2e)$.
3. Sort the following functions in increasing order of asymptotic growth:
   $$n^{\log n}, 2^n, n^n, 2^{\sqrt{\log n}}, 2^{n^2}, e^{2^n}, n, 2^{e^n}, n^e, \sqrt{n}, (\log n)^{\log n}.$$
   (For example, if you are given the functions n^2, n, and 2^n, the sorted list would be $n, n^2, 2^n$.) Show that for every pair of consecutive functions f, g in your list, f is $o(g)$.
4. Let T be the set of all positive integers that do not contain a 4 anywhere in their base-10 representation. For example, 7 and 335 are in T, but 5642 isn't. This question concerns the value of the sum
   $$S = \sum_{t \in T} \frac{1}{t} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{6} + \cdots + \frac{1}{13} + \frac{1}{15} + \cdots$$
   (a) Show that there are at most $8 \cdot 9^{d-1}$ numbers with d digits in T.
   (b) Use part (a) to show that the d-digit numbers contribute at most $8 \cdot 0.9^{d-1}$ to S.
   (**Hint:** What is the smallest d-digit number?)
   (c) Prove that $S < 100$. Can you prove that $S < 10$?