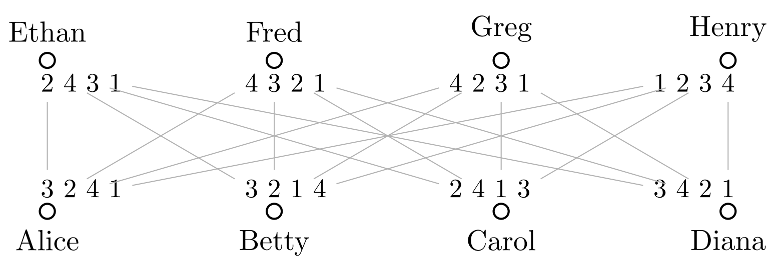
1. Do the following graphs exist? If yes, give an example. If no, prove that it doesn't.
   (a) A graph with 100 vertices of degree 3 and 3 vertices of degree 99.
   (b) A graph with 100 vertices of degree 2 and 2 vertices of degree 99.
   (c) A bipartite graph with 100 vertices of degree 2 and 2 vertices of degree 99.

2. A summer camp has children from *A*tlanta, *B*oston, and *C*hicago. The table entry $F(r, c)$ in row r and column c gives the average number of friends from city c that children from city r have made in the camp:

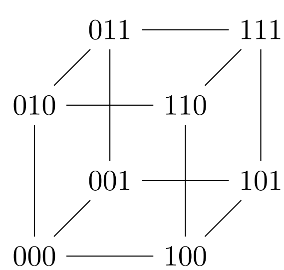
	A	B	C
A	2	?	4
B	3	5	1
C	6	2	3

   (a) Show that $F(r, c)/F(c, r)$ must equal (number of children from c)/(number of children from r).
   (b) Use part (a) to show that $F(A, B) \cdot F(B, C) \cdot F(C, A) = F(B, A) \cdot F(C, B) \cdot F(A, C)$.
   (c) Find the missing entry in the table.

3. Find stable matchings for the following preference lists with (a) boys proposing and girls choosing and (b) girls proposing and boys choosing.



4. The *hypercube* H_n of dimension n is the following graph on 2^n vertices: The vertices of H_n are all $\{0, 1\}$ strings of length n. There is an edge for any two vertices that differ in exactly one position. Here is a diagram of H_3:



   (a) Show that for every $n \geq 1$, H_n is a bipartite graph.
   (b) Show that for every $n \geq 1$, H_n has a perfect matching.
   (c) Now assume that n is odd and let G_n be the graph obtained by removing all vertices from H_n except those that have exactly $(n-1)/2$ zeroes or ones. Describe perfect matchings for the graphs G_3 and G_5.
   (d) (**Optional**) Prove that for every odd $n \geq 1$, G_n has a perfect matching. (**Hint:** Hall's theorem.)