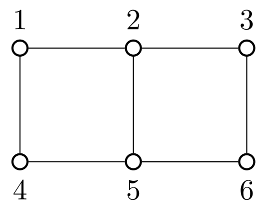
1. Are the following propositions about graphs true or false? Justify your answer. Specify your proof method.
   (a) Assume G is connected. Let G' be the graph obtained by removing an edge e from G. G' is connected if and only if e belongs to a cycle in G.
   (b) Assume G is connected. Let G' be the graph obtained by removing a vertex v and its incident edges from G. G' is connected if and only if v belongs to a cycle in G.
   (c) If every vertex in G belongs to a closed walk of odd length then there are at least as many edges as there are vertices in G.

2. Let G be the graph below. In this question you will count how many spanning trees G has.



   You will make use of the following auxiliary graph H: The vertices of H are the edges of G. A pair $\{e, f\}$ is an edge of H if removing edges e and f from G disconnects it.

   (a) Draw a diagram of H.
   (b) Argue that the number of spanning trees of G equals the number of vertex-pairs in H that *do not* form an edge.
   (c) Use parts (a) and (b) to count the number of spanning trees of G.

3. In this question you will work out vertex-disjoint paths for the following source-sink pairs in the Beneš network B_3. The sources are labeled `1` to `8` and the sinks are labeled `A` to `H` from top to bottom.

   `1E 2F 3D 4G 5B 6H 7C 8A`

   (a) For each source-sink pair above, determine whether the path should be routed through the top or through the bottom.
   (b) Route the top and bottom paths from part (a) recursively. Draw a diagram of the resulting eight vertex-disjoint paths.

4. Let G be the digraph whose vertices are the 125 3-digit numbers with digits 1, 2, 3, 4, 5, and (u, v) is an edge if $v - u$ equals 1, 10, or 100.
   (a) Show that G is acyclic.
   (b) What is the length of the longest path in G? Justify your answer.
   (c) Use part (b) to show that G must have an antichain of size ~~11~~ 10.
   (d) **(Optional)** Show that G has an antichain of size 19.
   (e) **(Optional)** Show that the vertices of G can be partitioned into 19 (vertex-disjoint) paths. Conclude that G cannot have an antichain of size 20.