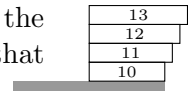
Practice Midterm 1

1. Prove that for every integer n there exists an integer k such that $|n^2 - 5k| \leq 1$. (**Hint:** What is n^2 mod 5?)
2. What is $1 + (1 + 2) + (1 + 2 + 3) + \cdots + (1 + 2 + 3 + \cdots + 1000)$?
3. Find a closed-form expression for the recurrence $f(n + 1) = 2f(n) + 2^{n-1}$, $f(1) = 0$.
4. You have overhang blocks 10, 11, up to n units long, one of each kind. They are stacked over the table from smallest to largest so that their left edges align. (See diagram for $n = 13$). Show that the configuration is not stable when n is sufficiently large.



Practice Midterm 2

1. Show that for every integer n, if $n^3 + n$ is divisible by 3 then $2n^3 + 1$ is *not* divisible by 3.
2. Let $f(n) = 1 + 1/3 + 1/5 + \cdots + 1/(2n - 1)$. Show that f is $\Theta(\log n)$.
3. An $n \times n$ plot of land (n is a power of two) is split in two equal parts by a North-South fence. The Western half is sold and the Eastern half is split in two equal parts by an West-East fence. The same procedure is applied to the remaining $(n/2) \times (n/2)$ plots until 1×1 plots are obtained (see $n = 4$ example). How many units of fence are used?

4. Sort these three functions in increasing order of growth: $\sqrt{n} \cdot \log n$, $n/\sqrt{\log n}$, $\sqrt{n \cdot \log n}$. For your sorted list f, g, h show that f is $o(g)$ and g is $o(h)$.

Practice Midterm 3

1. Bob has received from Alice the RSA ciphertext $c = 2$. The modulus is $n = pq$ with $p = 3$ and $q = 5$. The encryption key is $e = 3$.
   (a) Calculate Bob's decryption key d.
   (b) Decrypt Alice's message m.
2. What is the largest integer n for which
$$n \leq 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{9999}}?$$
3. Find a closed-form expression for the recurrence $f(n) = 3f(n - 1) + 4$, $f(0) = 0$.
4. Let $f(n)$ be the number of all length-n strings with symbols $\{\texttt{A}, \texttt{B}, \texttt{C}\}$ in which every `B` is immediately followed by a `C` (e.g., `BCAC` is counted but `ACAB` is not). Find the value of a for which $f(n)$ is $\Theta(a^n)$.