

Consider the propositions

$$P: \quad \forall x : F(x, \text{Alice})$$
$$Q: \quad \forall x : (F(x, \text{Alice}) \longrightarrow \forall y : F(x, y)).$$

Here x, y represent people (Alice, Bob, etc.) and $F(x, y)$ stands for “ x and y are friends”.

(a) Describe the meaning of P and Q in English.

Solution: P means “Everyone is friends with Alice.” Q means “Everyone who is friends with Alice is also friends with everyone else.”

(b) are P and Q logically equivalent? Justify your answer.

Solution: No. Consider the world that consists of Alice, Bob, and Charlie who are all mutual strangers. In this world P is false but Q is true.