Using induction, prove that for every $n \geq 1$, $1^2 + 2^2 + ... + n^2 > \frac{n^3}{3}$.

Solution: We proceed by induction on n.

Base Case: When $n = 1$ the left hand side is 1 and the right hand side is 1/3 so the inequality holds.

Inductive Step Assume $1^2 + 2^2 + ... + n^2 > \frac{n^3}{3}$. Then

$$\begin{aligned}
1^2 + 2^2 + ... + n^2 + (n+1)^2 &> \frac{n^3}{3} + (n+1)^2 && \text{by the inductive hypothesis} \\
&= \frac{n^3}{3} + \frac{3(n^2+2n+1)}{3} \\
&= \frac{n^3 + 3n^2 + 6n + 3}{3} \\
&> \frac{n^3 + 3n^2 + 3n + 1}{3} \\
&= \frac{(n+1)^3}{3}.
\end{aligned}$$

□