

Calculate $3^{33} \pmod{5}$.

Solution 1 using fast powering:

$$3^{33} \equiv 3 \cdot 3^{32} = 3 \cdot 9^{16} \equiv 3 \cdot (-1)^{16} = 3 \cdot 1^8 = 3 \pmod{5}.$$

Solution 2 using Fermat's little theorem: As $3^5 \equiv 3 \pmod{5}$, $3^4 \equiv 1 \pmod{5}$ so

$$3^{33} \equiv 3^{4 \cdot 8 + 1} = 3 \cdot (3^4)^8 \equiv 3 \cdot 1^8 = 3 \pmod{5}.$$