## Practice questions

1. Roll a die twice. Let $X$ be the larger number and $Y$ be the smaller number you rolled. Find the joint PMF of $X$ and $Y$, their marginal PMFs, and the expected value of $X+Y$.
2. A number between 1 to 100 is selected at random. You want to guess the number by asking questions of the type "Is the number equal to $x$ "? For every incorrect guess you lose $\$ 1$, and for the correct one you get $\$ 40$. You play this game until you make a correct guess. What is your expected profit?
3. On any given day between Monday and Saturday, the probability that you'll have a late snack is $20 \%$, independent on the other days. You'll have a late snack on Sunday if and only if you didn't have one in the previous six days. What is the expected number of snacks you'll be having?
4. Let $p$ be a number between 0 and 1. Toss a $p$-biased coin. If the coin comes up heads, toss a fair coin and report the outcome twice ( 1 for heads, 0 for tails). If the coin comes up tails, report the outcomes of two independent fair coin tosses. Show that the marginal PMFs of your two reports are the same for every $p$, but the joint PMFs are all different.
5. On a given day, your golf score takes values from the range 101 to 110 with probability 0.1 , independent of other days. To improve your score, you decide to play three times in a row and declare your score $X$ to be the minimum of those scores. Calculate the PMF of $X$. By how much has your expected score improved as a result of playing on three days? (Textbook problem 2.5.26)

## Additional ESTR 2002 questions

6. Alice picks a sequence $\left(x_{1}, x_{2}, x_{3}\right)$ of 3 numbers from the set $S=\{-1,0,1\}$. Bob picks two positions $i<j$ in the sequence. Alice and Bob simultaneously reveal their choices to one another and $s$ dollars are transferred from Alice to Bob, where $s$ is the unique value in $S$ that is congruent to $x_{j}-x_{i}$ modulo 3. For example, if Alice picks $(1,-1,1)$ and Bob picks $2<3$ then Alice pays Bob $x_{3}-x_{2}=1-(-1) \equiv-1$ dollars, that is she earns one dollar from Bob.
Alice's strategy is to choose her sequence at random so that her expected earnings are maximizes, regardless of Bob's choice. How much probability should she assign to each sequence?
7. The hot hand paradox is the belief that if your favorite sports team is on a "winning streak" then it is more likely to win the next game. For example, in this sequence of 38 wins and losses

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there are 12 consecutive wins. Was the team on a winning streak?
The Premier League has 20 football teams that play 38 rounds of matches. Let $S$ be the length of the longest winning streak in the league. Come up with a probability model and estimate the probability mass function, the expected value, and the standard deviation of $S$. I suggest that you start with a simple model that you can simulate (for instance, disregard draws), and then move to an even simpler one that you can analyze.

