1. Let $X, Y, Z$ be independent $\operatorname{Binomial}\left(2, \frac{1}{2}\right)$ random variables.
(a) What is the conditional PMF of $X$ conditioned on $X \neq Z$ ?
(b) Are $X$ and $Y$ independent conditioned on $(X \neq Z)$ And $(Y \neq Z)$ ?
2. Alice and Bob decide to meet somewhere. Alice's arrival time $A$ is uniform between 12:00 and 12:45. Bob's arrival time $B$ is uniform between 12:15 and 1:00. Their arrival times are independent.
(a) Let $f_{A-B}$ be the PDF of $A-B$. What is $f_{A-B}(0)$ ?
(b) What is the probability that Bob arrives before Alice?
3. Let $Y=A X+B$ where $A, B, X$ are independent $\operatorname{Normal}(0,1)$ random variables.
(a) What is $\operatorname{Var}[\mathrm{E}[Y \mid X]]$ ?
(b) What is $\mathrm{E}[\operatorname{Var}[Y \mid X]]$ ?
4. Boys and girls arrive independently at a meeting point at a rate of one boy per minute and one girl per minute, respectively. Let $T$ be the first time at which both a boy and a girl have arrived.
(a) Find the cumulative distribution function (CDF) of $T$.
(b) What is the expected value of $T$ ? (Hint: You don't have to use calculus.)
5. A deck of cards is divided into 26 pairs. Let $X$ be the number of those pairs in which both cards are of the same suit. (A deck of cards has 4 suits and each suit has 13 cards.)
(a) What is the expected value of $X$ ?
(b) What is the variance of $X$ ?
(c) Is the probability that $X=0$ more or less than $20 \%$ ? Justify your answer.
