Each question is worth 10 points. Please explain your solution clearly and concisely.

1. Show that $\sqrt{2} + \sqrt{3}$ is an irrational number.
2. You have a 6 litre, a 10 litre, and a 45 litre jug, and a water source. Can you measure 1 litre using the pouring rules from Lecture 4?
3. Show that in every graph, the sum of the squares of the degrees of all the vertices is an even number.
4. Show that in every digraph in which there is no source there are two vertices of the same in-degree.
5. Show that $1 + 1/4^2 + 1/7^2 + \cdots + 1/(3n+1)^2$ is $\Theta(1)$.
6. How many five-card poker hands are there that have the same number of kings and aces?
7. Let p be a polynomial of the form $p(x) = ax^4 + bx^3 + x^2$ over $\mathbb{F}_q$, where q is a prime number. Show that p has at most three zeros.
8. Alice comes up with a circular seating arrangement for n guests at a round table. Show that Bob needs to ask Alice $\Omega(n \log n)$ yes/no questions in order to figure out the arrangement with certainty.