1. Write the proposition "There is a cycle of length 3 " using logical connectives and quantifiers. Use symbols $x, y, z$ for vertices and $E(x, y)$ for " $\{x, y\}$ is an edge."
2. Show that for all positive integers $m$ and $n$, if $\operatorname{gcd}\left(m^{2}, n^{2}\right)$ is odd then $m$ and $n$ are not both even.
3. Use induction to show that for every $n \geq 1$, the $(n+1) \times n$ grid can be tiled using two sets of the following tiles: $1 \times 1,1 \times 2, \ldots, 1 \times n$.
4. You start with the numbers $1,2,3,4$. At each step, you take any three numbers $a, b, c$ and replace them with $(a+b) / 2,(b+c) / 2,(c+a) / 2$. The fourth one stays the same. Can you ever get $2,3,4,5$ ?
5. Give a stable matching for the following preferences. Show that there is no rogue couple among the six man-woman pairs that are not matched to one another.

6. You are given a graph with 9 men and 9 women as vertices and all possible 81 man-woman pairs as edges. Let $\Xi$ be any matching in this graph. Remove the edges in $\Xi$ (but not the vertices.) Show that the remaining graph has a perfect matching.
