

Each question is worth 10 points. Please explain your solution clearly and concisely.

1. Is the following deduction rule valid?

$$\frac{P \longrightarrow Q \quad P \longrightarrow R}{Q \text{ OR } R}$$

2. Let a and b be real numbers. Show that if a is rational and ab is irrational, then b is irrational.
3. Show that for every $\ell \geq 3$, a cycle of length ℓ has a perfect matching if and only if ℓ is even.
4. Show that for every integer $n \geq 1$, $1 + 1/4 + 1/9 + \cdots + 1/n^2 \leq 2 - 1/n$.
5. The multiplicative inverse of 3 modulo 23 is 8. The multiplicative inverse of 12 modulo 23 is 2. What is the multiplicative inverse of 13 modulo 23? Explain your reasoning.
6. Show that for every $n \geq 2$, a 6 by n grid can be covered using L-shaped tiles. (Each L-shaped tile occupies 3 squares and the tiles must not overlap.)