Practice questions

Clearly describe the sample space, the events of interest, and the probability model whenever appropriate.

1. Alice, Bob, and Charlie hold a lucky draw for two tickets to a concert with the following odds:
   - The probability that Alice gets one of the tickets is 60%.
   - The probability that Bob gets one of the tickets is 70%.

   What is the probability that Alice and Bob both get tickets?

2. A machine produces parts that are either good (90%), slightly defective (2%), or obviously defective (8%). Produced parts get passed through an automatic inspection machine, which is able to detect any part that is obviously defective and discard it. What is the quality of the parts that make it through the inspection machine and get shipped?

3. You flip a fair coin five times independently. Let H_1 be the event that the first flip is a head and M be the event that a majority (at least 3 out of 5) of the flips are heads. Calculate (a) $\mathrm{P}(M)$; (b) $\mathrm{P}(M|H_1)$; (c) $\mathrm{P}(H_1|M)$.

4. A bin contains 3 white balls and 5 black balls. Alice and Bob take turns drawing balls from the bin without replacement until a white ball is drawn. Assuming Alice goes first, what is the probability that she gets the white ball?

Additional ESTR 2018 questions

5. Benford's law is a probability model over the sample space $\{1, 2, \ldots, 9\}$ with $\mathrm{P}(\{d\}) = \log_{10}(d + 1) - \log_{10} d$. It describes the probability of the leading (most significant) digit in real-life numerical data like accounting records. Benford's law predicts, for example, that the leading digit is a 1 about 30% of the time. Test this hypothesis on some data sets of your choice.

6. In a fair contract for an event E, you pay $\mathrm{P}(E)$ dollars before the experiment, collect 1 dollar if the event happens and 0 dollars if it doesn't. Show that the exactly one of the following two statements must be true:
   - $\mathrm{P}(\cdot)$ satisfies the axioms of probability
   - There exists a combination of fair contracts that you can buy and sell for which regardless of the outcome you come out ahead by some positive amount. (Assume that you are allowed to buy or sell any fractional amount of fair contracts for any event of your choice.)

   Now suppose you have one dollar, and buying or selling a contract costs 0.01 dollars. By "how much" can the axioms of probability be violated without allowing certain profit regardless of outcome?