ENGG 2760A / ESTR 2018: Probability for Engineers — Homework 3

The Chinese University of Hong Kong, Fall 2020

Practice questions

1. There are 5 red balls, 5 blue balls, and 5 green balls in a bin. You draw two balls from a bin. What is the probability that

   (a) Ball 2 is red?

   (b) Ball 2 is red given that ball 1 is red, if the balls are drawn *with* replacement?

   (c) Ball 2 is red given that ball 1 is red, if the balls are drawn *without* replacement?

   (d) Ball 2 is not blue given that ball 1 is red, if the balls are drawn *without* replacement?

   (e) Ball 2 is red given that ball 1 is not blue, if the balls are drawn *without* replacement?

2. Roll three 6-sided dice. Let E_{12} be the event that the first face is *the same as* the second face. Define E_{23} and E_{13} analogously. Determine which of the following statements are true:

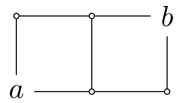
   (a) Any two of the three events E_{12}, E_{23}, E_{13} are independent.

   (b) E_{12}, E_{23}, and E_{13} are independent.

   (c) E_{12} and E_{23} are independent conditioned on E_{13}.

3. If Alice flips 10 coins and Bob flips 9 coins, what is the probability that Alice gets more heads than Bob? (**Hint:** Use conditioning. You may want to work out a smaller example first.)

4. Computers a and b are linked through seven cables as in the picture. Each cable fails with probability 10% independently of the others. Let C be the event "there is a connection between a and b" and F be the event "the middle vertical cable fails".



   (a) What is the probability of C given F?

   (b) What is the probability of C given F^c?

   (c) What is the probability of C?

Additional ESTR 2018 questions

5. You have collected the following statistics on the fraction of smokers and drinkers that suffer from medical condition X among 1000 randomly tested individuals:

smokes	drinks	has X	number of cases
yes	yes	yes	3
yes	yes	no	27
yes	no	yes	7
yes	no	no	63
no	yes	yes	2
no	yes	no	68
no	no	yes	38

   Does drinking affect condition X?

6. Can there be four events E_1, E_2, E_3, E_4 so that every pair E_i, E_j is independent but every triple E_i, E_j, E_k is not (i, j, k are distinct indices)?

   More generally, suppose you are given a set $\mathcal{I}$ consisting of *subsets* of $\{1, \ldots, n\}$. Under which conditions on $\mathcal{I}$ can there exist a sample space Ω and events $E_1, \ldots, E_n$ such that for every set of indices I, the events $E_i \colon i \in I$ are independent when $I \in \mathcal{I}$ and not independent otherwise?