Practice questions

1. Alice rolls three 3-sided dice. Calculate the PMFs and the expected values of

   (a) The maximum of the three rolls.
   (b) The minimum of the three rolls.
   (c) The average of the three rolls.
   (d) **(Optional)** The median of the three rolls.

2. Suppose the number of school bus arriving at the Sir Run Run Shaw Hall in any time interval is a Poisson random variable, with a rate of 1 bus in 5 minutes.

   (a) What is the probability that no bus arrives in an interval of 30 minutes?
   (b) What is the probability that there are at least 5 buses in an interval of 10 minutes?

3. Alice can't find her expensive sweater. She estimates that there is a 30% chance that she left it at the café and a 40% chance that she left it at the shop (and that it is lost with the remaining probability). The distances between her home, the café, and the shop are given below. On her trip to find the sweater, in which order should she visit the venues so as to minimize her expected round-trip walking distance?
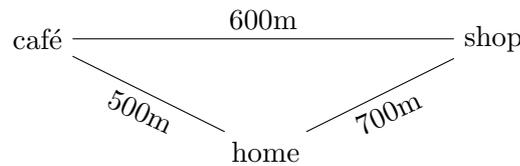
4. Calculate the PMFs of the following random variables:

   (a) The first time X at which both the patterns TH and HT have appeared in a sequence of fair coin flips. For example, $X = 6$ for the sequence HHTTTH.
   (b) The first time Y at which all three face values have appeared in a sequence of rolls of a fair 3-sided die. For example, $Y = 6$ for the sequence 232231.

Additional ESTR 2018 questions

5. Alice picks a sequence (x_1, x_2, x_3) of 3 numbers from the set $S = \{-1, 0, 1\}$. Bob picks two positions $i < j$ in the sequence. Alice and Bob simultaneously reveal their choices to one another and s dollars are transferred from Alice to Bob, where s is the unique value in S that is congruent to $x_j - x_i$ modulo 3. For example, if Alice picks $(1, -1, 1)$ and Bob picks $2 < 3$ then Alice pays Bob $x_3 - x_2 = 1 - (-1) \equiv -1$ dollars, that is she earns one dollar from Bob.

   Alice's strategy is to choose her sequence at random so that her *expected* earnings are maximizes, regardless of Bob's choice. How much probability should she assign to each sequence?

6. The *hot hand paradox* is the belief that if your favorite sports team is on a "winning streak" then it is more likely to win the next game. For example, in this sequence of 38 wins and losses

   LLLWWWLWLLLLWWLLLLLWLLWWLWWLWLLWLLLWWLWLLLWLLWWWWWWWWWWWLLLLW

there are 12 consecutive wins. Was the team on a winning streak?

The Premier League has 20 football teams that play 38 rounds of matches. Let S be the length of the longest winning streak in the league. Come up with a probability model and estimate the probability mass function and the expected value of S. I suggest that you start with a simple model that you can simulate (for instance, disregard draws), and then move to an even simpler one that you can analyze.