Practice questions

1. A box contains 7 red and 7 blue balls. Two balls are drawn without replacement. If they are the same color, then you win \$1; if they are different colors, then you lose \$1.
   (a) What is the expected value of the amount you win?
   (b) What is the variance of the amount you win?

2. Roll a 4-sided die twice. Let X and Y be the minimum and maximum of the two rolls, respectively. Find the joint PMF of X and Y, their marginal PMFs, and the expected value of $X + Y$.

3. Let p be a number between 0 and 1. Toss a p-biased coin. If the coin comes up heads, toss another fair coin and report the outcome twice (1 for heads, 0 for tails). If the coin comes up tails, report the outcomes of two independent fair coin tosses. Show that the marginal PMFs of your two reports are the same for every p, but the joint PMFs are all different.

4. 100 balls are tossed at random into 100 bins. Each ball is equally likely to land in any of the bins, independently of the other balls.
   (a) What is the expected number of bins that receive exactly one ball?
   (b) What is the expected number of balls that are not alone in their bin?

Additional ESTR 2018 questions

5. The Poisson random variable measures the number of successes in many *independent* trials, each with small success probability. In some cases, the Poisson random variable gives an approximate answer even if the trials are not completely independent. It is not so easy to explain the general conditions under which such "Poisson approximation" works but here is one example.

   In Lecture 1 we talked about a system with n antennas on a line, where each of the antennas can be functional or defective. The system fails whenever two *consecutive* antennas are defective. Assume the defects are independent and each happens with probability p.

   (a) Let P_n be the probability that the system fails. Derive a recurrence for P_n. Solve this recurrence for $n = 10$ and $p = 0.1$.
   (b) Say a *failure* happens at position i if the i-th and $(i + 1)$-st antenna are both defective. Argue that the events "A failure happens at position i", $i = 1, \dots, n - 1$ are not independent.
   (c) Calculate the expected number of failures μ.
   (d) Let N be a Poisson(μ) random variable. Calculate $P(N \neq 0)$ and compare to part (a).

6. In Lecture 1 we also argued that a particle that moves at random one step left or right each time eventually revisits its origin. On average how many times will the particle visit the origin in the first 100 steps of its walk? Can you guess a rough formula for the expected number of returns to the origin in n steps when n is large?