Practice questions

1. The joint PDF of X and Y is

$$f_{X,Y}(x,y) = \begin{cases} C(x+y+1)y, & \text{if } 0 \le x \le 2, 0 \le y \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

   Find (a) the value of C and (b) The conditional PDF $f_{Y|X}(y|x)$.

2. Alice and Bob agree to meet. Alice's arrival time A is uniform between 12:00 and 12:45 and Bob's arrival time B is uniform between 12:15 and 13:00. Let E be the event "Alice and Bob arrive within 30 minutes of one another".

   (a) What is $\mathrm{P}(E)$ assuming A and B are independent?

   (b) If you don't know the joint PDF of A and B, how large can $\mathrm{P}(E)$ be?

   (c) **(Optional)** If you don't know the joint PDF of A and B, how small can $\mathrm{P}(E)$ be?

3. Raindrops hit the ground at a rate of 1 per second. An observatory has a raindrop sensing equipment. A signal is received by the computer with a maximum delay of 1 second after sensing a raindrop, with all delays equally likely. Find

   (a) The joint PDF of the time T of the first raindrop and the time S of the signal reception.

   (b) The marginal PDF of S.

   (c) The conditional PDF of T given S.

4. Here is a way to solve Buffon's needle problem without calculus. Recall that an ℓ inch needle is dropped at random onto a lined sheet, where the lines are one inch apart.

   (a) Let A be the number of lines that the needle hits. Let B be the number of times that a polygon of perimeter ℓ hits a line. Show that $\mathrm{E}[A] = \mathrm{E}[B]$. (**Hint:** Use linearity of expectation.)

   (b) Assume that $\ell < \pi$. Calculate the expected number of times that a circle of perimeter ℓ hits a line.

   (c) Assume that $\ell < 1$. Use part (a) and (b) to derive a formula for the probability that the needle hits a line. (**Hint:** The number of hits is a Bernoulli random variable.)