

Questions

1. Raindrops are falling at the rate of 1 drop per second.
 - (a) Use Markov's inequality to argue that the probability of getting more than 120 raindrops within a minute is at most 50%.
 - (b) Use Chebyshev's inequality to argue that the probability of getting more than 120 raindrops within a minute is in fact at most 2%.
2. Alice is mailing letters to solicit donations from CUHK alums. From past experience she knows that 30% of the alums make a 500 dollar donation, and 10% of the alums make a 1,000 dollar donation. Use the central limit theorem to estimate the number of letters Alice should mail to meet a 50,000 dollar donation target with probability 90%.
3. The following exam statistics are posted on the course website:

section	no. students	average grade	std. dev.
A	30	65	5
B	20	70	10

what can you say about the number of students whose exam grade was 30 or below?

Estimate the quantity of your interest using (a) Markov's inequality, (b) Chebyshev's inequality and (c) the Central Limit Theorem. Explain the assumptions you are making about the probability model (if any).

4. There are 6 computers. Every pair of computers connects with probability 10%, independently of the other pairs. Say a computer is *isolated* if it didn't connect to any of the other computers. Let N be the number of isolated computers.
 - (a) Calculate the expected value of N .
 - (b) Calculate the variance of N .
 - (c) Argue that the probability that at least one computer is isolated is 70% or more.