Each question is worth 10 points. Explain your answers clearly.

1. The joint probability density function of the lifetimes X and Y of two connected components in a machine is
$$f_{X,Y}(x,y) = \begin{cases} xe^{-x(1+y)}, & x \geq 0, y \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$
   (a) What is the probability that the lifetime X of the first component exceeds 3?
   (b) Are X and Y independent? Justify your answer.

2. A radio station gives a gift to the third caller who knows the birthday of the radio talk show host. Each caller has a 0.7 probability of guessing the host's birthday, independently of other callers.
   (a) What is the probability mass function of the number of calls necessary to find the winner?
   (b) What is the probability that the station will need five or more calls to find a winner?

3. Alice sends a message a that equals -1 or 1. Bob receives the value B which is a Normal random variable with mean a and standard deviation 0.5. Bob guesses that Alice sent 1 if $B > 0.5$, that Alice sent -1 if $B < -0.5$, and declares failure otherwise (when $|B| \leq 0.5$).
   (a) What is the probability that Bob declares failure?
   (b) Given that Bob didn't declare failure, what is the probability that his guess is correct?

4. The number of people who enter an elevator on the ground floor is a Poisson random variable with mean 10. There are 20 floors above (not including) the ground floor and each person is equally likely to get off on any one of them, independently of all others.
   (a) What is the probability p that the elevator doesn't stop on the seventh floor?
   (b) What is the expected number of stops that the elevator will make?
   (Express the answer in terms of p in case you didn't complete part (a).)

5. 500 balls are drawn without replacement from a bin with 600 black balls and 400 white balls.
   (a) What is the expected number of black balls drawn?
   (b) What is the variance of the number of black balls drawn?
   (c) Is the probability you drew fewer than 200 black balls more than 2%? Justify your answer.

x :	0	0.5	1	1.5	2	2.5	3	3.5
$\mathrm{P}(X \leq x)$:	0.5	0.6915	0.8413	0.9331	0.9772	0.9938	0.9987	0.9998

CDF of a Normal$(0,1)$ random variable X