Each question is worth 10 points. Explain your answers clearly.

1. X and Y are independent random variables, both with the following PMF:

x	1	2	4
$f(x)$	1/3	1/3	1/3

   (a) Find the PMF of $X + Y$.

   (b) Are X and $X + Y$ independent? Justify your answer.

2. The number of cars behind a traffic light at the time it turns green is a Poisson random variable X with mean 1. The number of cars that cross the green light is $\min\{X, 3\}$.

   (a) Find the PMF of the number of cars that cross the (green) light.

   (b) The light turns green 50 times within the hour. Is the probability that more than 100 cars cross within the hour larger or smaller than 50%? Justify your answer.

3. Alice and Bob independently arrive at the bus stop at a uniformly random time between 8 and 9. There are buses at 8.15, 8.30, and 9.

   (a) What is the probability that they catch the same bus?

   (b) Given that Bob did't run into Alice on the 8.30 bus, what is the probability that Alice caught the 8.15 bus?

4. The body weight of a random person is a normal random variable with mean 60kg and standard deviation 10kg. The carrying capacity of an elevator is 600kg. If nine people enter the elevator, what is the probability that the weight limit is exceeded? Assume their weights are independent.

5. A deck of cards is divided into 26 pairs. Let X be the number of those pairs in which both cards are of the same suit. (A deck of cards has 4 suits and each suit has 13 cards.)

   (a) What is the expected value of X?

   (b) What is the variance of X?