ENGG 2760A / ESTR 2018: Probability for Engineers Tutorial

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A set vs the size of the set :

Set : a collection of objects, which are the elements of these

The size of set: the number of elements in the set. $|S|$

Example. $S = \{1, 2, \cdots, 6\}$ $|S| = 6$

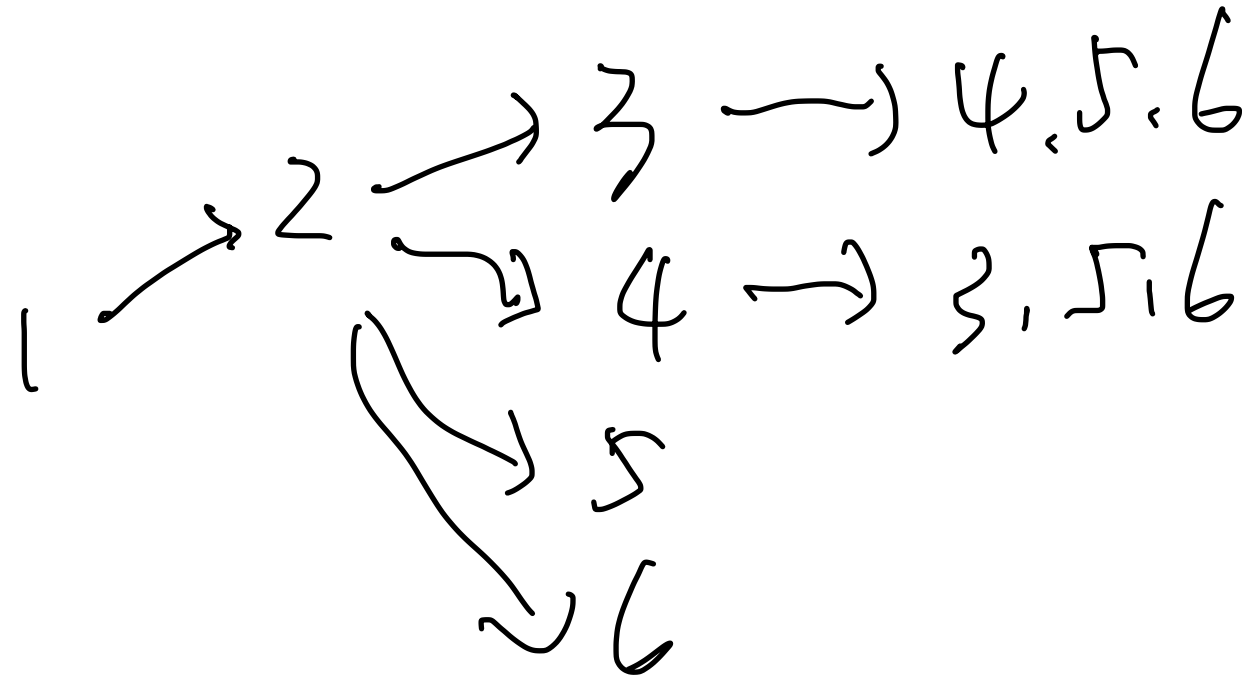
$S = \{H, T\}$ $|S| = 2$

1. (a)

$$\Omega = \{1, 2, 3, \cdots, 6\}^4$$

$$A = \{(f_1, f_2, f_3, f_4) : \text{all different}\}$$

f_i : the face value of i-th dice



$$\therefore |A| = 6 \times 5 \times 4 \times 3 = 360$$

$\Omega = \{1, 2, 3, 4, 5, 6\}^4 \rightarrow A = \{(1,3,5,6), (1,3,6,5)$
$(1,5,3,6) \cdots\}$

$$|A| = \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360$$

(b) $\Omega = \{1, 2, 3, 4, 5, 6\}^4$

$$B = \{(f_1, f_2, f_3, f_4) : f_1 < f_2 < f_3 < f_4\}$$

$$B = \{(1, 3, 5, 6)\}$$

$$|B| = \binom{6}{4} = \frac{6!}{4!\,2!} = \frac{6 \times 5}{2} = 15$$

2. Step 1: specify all possible outcomes
   Step 2: identify events of interest
   Step 3: Assign probabilities
   Step 4: calculate

Step1: $\Omega = \{(x_1, x_2, x_3) : x_i \text{ draw from 20 balls}, i = 1,2,3\}$

$|\Omega| = \binom{20}{3}$

Step2: $A = \{(x_1, x_2, x_3) : x_i \text{ from 10 black balls}, i = 1,2,3\}$

$|A| = \binom{10}{3}$

Step3: Assume outcomes equally likely

$$P(A) = \frac{|A|}{|\Omega|} = \frac{\binom{10}{3}}{\binom{20}{3}} = \frac{10!}{7!3!} \cdot \frac{17!3!}{20!}$$

$$= \frac{10 \times 9 \times 8}{20 \times 19 \times 18}$$

3.(a) $|\Omega| = \binom{100}{30, 35, 35} = \frac{100!}{30! \cdot 35! \cdot 35!}$

A: Alice & Bob in 30-student tutorial

$$|A| = \binom{98}{28, 35, 35} = \frac{98!}{28! \cdot 35! \cdot 35!}$$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{\binom{98}{28,35,35}}{\binom{100}{30,35,35}} = \frac{98!}{28!\,35!\,35!} \cdot \frac{30!\,35!\,35!}{100!}$$

$$= \frac{30 \times 29}{100 \times 99}$$

$\binom{100}{30,35,35}$

100 students

$\hookrightarrow$ 30 $\rightarrow$ 1 tutorial

remaining 70 students

$\hookrightarrow$ 35 $\rightarrow$ 2 tutorial

remaining 35 $\rightarrow$ 3 tutorial

$$\binom{100}{30}\binom{70}{35}\binom{35}{35}$$

$$= \frac{100!}{30!\,70!} \cdot \frac{70!}{35!\,35!} \cdot \frac{35!}{35!} = \frac{100!}{30!\,35!\,35!}$$

3.(b) $|\Omega| = \binom{100}{30,35,35}$

$A = A_1 \cup A_2 \cup A_3$

A_i : Alice & Bob assigned to ith tutorial

A_2, A_3 ?

$|A_2| = |A_3| = \binom{98}{30,33,35}$

$|A| = |A_1| + |A_2| + |A_3| = \binom{98}{28,35,35} + 2\times\binom{98}{30,33,35}$

$P(A) = \frac{|A|}{|\Omega|} = \frac{\frac{98!\times30\times29}{28!\cdot35!\cdot35!} + 2\times\frac{98!\times35\times34}{30!\times33!\times35!}}{\frac{100!}{30!\cdot35!\cdot35!}}$ $\frac{100\times99}{}$

$= \frac{30\times29 + 2\times35\times34}{100\times99}$

4. A : a sum of 11 $\{(x_1,x_2,x_3): x_1+x_2+x_3=11\}$

B : a sum of 12 $\{(x_1,x_2,x_3): x_1+x_2+x_3=12\}$

Q: $P(A) \overset{?}{=} P(B)$

$\Omega = \{1,2,\cdots,6\}^3$ $|\Omega| = 6^3$

Assume outcomes equally likely

$P(A) = |A|/6^3$ $P(B) = |B|/6^3$

$\{(1,x_2,x_3): x_2+x_3=10\}$

Q $\Rightarrow$ $|A| \overset{?}{=} |B|$ $A = A_1 \cup A_2 \cup A_3 \cdots \cup A_6$

A : $A_1, A_2, \cdots, A_6$. A_i means the face value of first dice is i

second + third = 11 − i

B : $B_1, B_2, \cdots, B_6$. $B_i \mapsto \{(1,x_2,x_3): x_2+x_3=11\}$

second + third = 12 − i $B = B_1 \cup B_2 \cdots \cup B_6$

$|A_1| = |B_2|$, $A_1 = \{(1,x_2,x_3): x_2+x_3=10\}$

$B_2 = \{(2,x_2,x_3): x_2+x_3=10\}$

$|A_2| = |B_3|$, $|A_3| = |B_4|$, $|A_4| = |B_5|$, $|A_5| = |B_6|$

$|A| = |A_1| + |A_2| + \cdots + |A_6|$

$|B| = |B_1| + |B_2| + \cdots + |B_6|$

$|A| ? |B|$

Compare $|A_6|$ and $|B_1|$

A_6: remaining add up $5 = 1+4, 4+1, 2+3, 3+2$

B_1: $\sim$ add up $11 = 5+6, 6+5$

$|A_6| = 4 > |B_1| = 2$ $A_6 = \{(6, x_2, x_3) : x_2 + x_3 = 5\}$

$\Rightarrow |A| > |B|$

$\Rightarrow p(A) > p(B)$

A: the sum of 3 dice is 11

Q: P(A)

$A: \{(x_1, x_2, x_3) : x_1+x_2+x_3=11\}$

$11 = 6+4+1$ · ×3!

$= 6+3+2$ ×3!

$= 5+5+1$ ×3

$= 5+4+2$ ×3!

$= 5+3+3$ ×3

$= 4+4+3$ ×3

~~$= 3+6+2$~~

combination → order

6 + 6 + 3 + 6 + 3 + 3 = 27

x_1, x_2, x_3 (x_i means the face value of the i-th order)

(5,5,1)

(5,1,5)

(1,5,5)

3

(5,4,2) (5,2,4)

(4,5,2) (4,2,5)

(2,5,4) (2,4,5)

3-permutation $= 3!$