## Practice questions

Describe the sample space, the events of interest, and the probability model where appropriate.

1. Alice, Bob, and Charlie hold a lucky draw for two tickets to a concert with the following odds:

- The probability that Alice gets one of the tickets is $60 \%$.
- The probability that Bob gets one of the tickets is $70 \%$.

What is the probability that Alice and Bob both get tickets?
2. Alice flips seven fair coins. Let $H$ be the event that the last flip is a head and $A$ be the event that at least one flip is a head. Calculate (a) $\mathrm{P}\left(A^{c}\right) ;$ (b) $\mathrm{P}(H \mid A) ;(\mathrm{c}) \mathrm{P}(A \mid H)$.
3. The Los Angeles Lakers and the Boston Celtics play one game in each city. Each team wins their home game with $70 \%$ probability. There is a $40 \%$ probability that both win their home games. The Lakers win their home game. What is the probability that they win in Boston?
4. There are 6 red balls and 1 blue ball. Each ball is randomly placed in one of two bins.
(a) What is the probability that the bin with the larger number of balls contains $k$ balls $(k \in\{4,5,6,7\}) ?$
(b) What is the probability that the bin with the larger number of balls contains 3 red balls and 1 blue ball?

## Additional ESTR 2018 questions

5. Let $A$ be the event "Peter will have installed a home alarm by the end of next year" and $B$ be the event "Peter's home will be burglarized by the end of next year." The psychologists Amos Tversky and Daniel Kahneman carried out a survey in which 131 out of 162 people said that $\mathrm{P}(A \mid B)>\mathrm{P}\left(A \mid B^{c}\right)$ and $\mathrm{P}(B \mid A)<\mathrm{P}\left(B \mid A^{c}\right)$. Prove that this is impossible. How do you explain the results of the survey? As a possible project you can read up on this and other experiments in which human intuition is at odds with probability theory. [Exercise 2.14 in Blitzstein-Hwang textbook]
6. In a contract for an event $E$, Alice pays $\mathrm{P}(E)$ dollars before the experiment, collects $\$ 1$ if the event happens and $\$ 0$ if it doesn't. Suppose Bob thinks there is a $60 \%$ probability that tomorrow will be sunny and a $30 \%$ probability that it won't. Alice can buy a "sunny" contract from Bob for $\$ 60$ and a "not sunny" contract for $\$ 30$. She will be $\$ 10$ ahead no matter the outcome.
Show that exactly one of these two statements must be true:

- Bob's beliefs satisfy the axioms of probability
- There exists a combination of (buy or sell) contracts by which Alice can profit from Bob regardless of outcome. Assume that a contract for any event can be sold or bought.

Now suppose that for each dollar of contract Alice has to pay a tax of one cent. Can you come up with a world in which Bob's beliefs are inconsistent with the axioms yet Alice cannot profit with certainty? Can you design an algorithm that takes Bob's beliefs (for all possible events) as input and determines if there are profitable contracts for Alice?

