- 1. Roll a 4-sided die twice. Let X be the larger number and Y be the smaller number you rolled. Find (a) the conditional PMF of X given Y and (b) E[X|Y = y] for y = 1, 2, 3, 4.
- 2. Cup 1 contains three \$1 coins. Cup 2 contains a \$1 coin, a \$2 coin, and a \$5 coin. Alice chooses a random cup, takes out two coins, and gives the second coin to Bob.
 - (a) How many dollars does Bob expect to gain?
 - (b) Bob sees that the first coin out is a \$1 coin. Did his expected gain increase?
- 3. Express X and Y below as 1 + A + B for some Geometric random variables A and B. Use this formula to find E[X] and E[Y]. Are A and B independent? If yes, find Var[X] and Var[Y].
 - (a) The first time X at which both a TH and a HT have appeared in a sequence of coin flips.
 - (b) The first time Y at which 1, 2, and 3 have all appeared in a sequence of 3-sided die rolls.
- 4. You roll a six-sided die and the value is X_1 . You can either cash in X_1 dollars, or choose to roll again, in which case you cash in the value X_2 of the second roll in dollars.
 - (a) For which values of X_1 should you roll again in order to maximize your expected utility?
 - (b) What is your expected utility for the strategy in part (a)?
 - (c) You can now roll a third time if you are unhappy with X_2 . How does this change your answer in (a) and (b)? (*Hint:* What is your maximum expected utility given $X_1 = x_1$?)
 - (d) (Extra credit) What is your maximum expected utility if you can roll up to t times?

[Based on Blitzstein-Hwang exercise 4.4]

Additional ESTR 2018 questions

- 5. Let X be the number of times a 6-sided die is rolled until a 6 appears. Let A be the event that all rolls before the first 6 are even numbers. This question is about the value E[X|A]. [due to E. Mossel]
 - (a) Let Y be the number of times a 3-sided die with face values 2, 4, 6 is rolled until a 6 appears. What is E[Y]? Do you think that E[X|A] and E[Y] should be the same?
 - (b) Calculate $P((X = x) \cap A)$ for x = 1, 2, ...
 - (c) Calculate the conditional PMF P(X = x|A) and use it to obtain E[X|A].
 - (d) Are the answers in parts (a) and (c) the same? Why not?