1. A point is chosen uniformly at random inside a circle with radius 1 . Let $X$ be the distance from the point to the centre of the circle. What is the (a) CDF (b) PDF (c) expected value and (d) variance of $X$ ? [Adapted from textbook problem 3.2.7]
2. Bob's arrival time at a meeting with Alice is $X$ hours past noon, where $X$ is a random variable with PDF

$$
f(x)= \begin{cases}c x, & \text { if } 0 \leq x \leq 1 \\ 0, & \text { otherwise }\end{cases}
$$

(a) Find the value of the constant $c$.
(b) What is the probability that Bob arrives by 12.30 ?
(c) What is the expected time of Bob's arrival?
(d) Given that Bob hasn't arrived by 12.30 , what is the probability that he arrives by 12.45 ?
(e) Given that Bob hasn't arrived by 12.30, what is the expected time of Bob's arrival?
3. The joint PDF of $X$ and $Y$ is

$$
f_{X, Y}(x, y)= \begin{cases}C(x+y+1) y, & \text { if } 0 \leq x \leq 2,0 \leq y \leq 2 \\ 0, & \text { otherwise }\end{cases}
$$

Find (a) the value of $C$ and (b) The conditional PDF $f_{Y \mid X}(y \mid x)$.
4. Alice and Bob agree to meet. Alice's arrival time $A$ is uniform between 12:00 and 12:45 and Bob's arrival time $B$ is uniform between 12:15 and 13:00. Let $E$ be the event "Alice and Bob arrive within 30 minutes of one another".
(a) What is $\mathrm{P}(E)$ assuming $A$ and $B$ are independent?
(b) If you don't know the joint PDF of $A$ and $B$, how large can $\mathrm{P}(E)$ be?
(c) (Optional) If you don't know the joint PDF of $A$ and $B$, how small can $\mathrm{P}(E)$ be?
5. (Optional) Here is a way to solve Buffon's needle problem without calculus. Recall that an $\ell$ inch needle is dropped at random onto a lined sheet, where the lines are one inch apart.
(a) Let $A$ be the number of lines that the needle hits. Let $B$ be the number of times that a polygon of perimeter $\ell$ hits a line. Show that $\mathrm{E}[A]=\mathrm{E}[B]$. (Hint: Use linearity of expectation.)
(b) Assume that $\ell<\pi$. Calculate the expected number of times that a circle of perimeter $\ell$ hits a line.
(c) Assume that $\ell<1$. Use part (a) and (b) to derive a formula for the probability that the needle hits a line. (Hint: The number of hits is a Bernoulli random variable.)

