- 1. A point is chosen uniformly at random inside a circle with radius 1. Let X be the distance from the point to the centre of the circle. What is the (a) CDF (b) PDF (c) expected value and (d) variance of X? [Adapted from textbook problem 3.2.7]
- 2. Bob's arrival time at a meeting with Alice is X hours past noon, where X is a random variable with PDF

$$f(x) = \begin{cases} cx, & \text{if } 0 \le x \le 1\\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the value of the constant c.
- (b) What is the probability that Bob arrives by 12.30?
- (c) What is the expected time of Bob's arrival?
- (d) Given that Bob hasn't arrived by 12.30, what is the probability that he arrives by 12.45?
- (e) Given that Bob hasn't arrived by 12.30, what is the expected time of Bob's arrival?
- 3. The joint PDF of X and Y is

$$f_{X,Y}(x,y) = \begin{cases} C(x+y+1)y, & \text{if } 0 \le x \le 2, 0 \le y \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Find (a) the value of C and (b) The conditional PDF $f_{Y|X}(y|x)$.

- 4. Alice and Bob agree to meet. Alice's arrival time A is uniform between 12:00 and 12:45 and Bob's arrival time B is uniform between 12:15 and 13:00. Let E be the event "Alice and Bob arrive within 30 minutes of one another".
 - (a) What is P(E) assuming A and B are independent?
 - (b) If you don't know the joint PDF of A and B, how large can P(E) be?
 - (c) (Optional) If you don't know the joint PDF of A and B, how small can P(E) be?
- 5. (Optional) Here is a way to solve Buffon's needle problem without calculus. Recall that an ℓ inch needle is dropped at random onto a lined sheet, where the lines are one inch apart.
 - (a) Let A be the number of lines that the needle hits. Let B be the number of times that a polygon of perimeter ℓ hits a line. Show that E[A] = E[B]. (Hint: Use linearity of expectation.)
 - (b) Assume that $\ell < \pi$. Calculate the expected number of times that a circle of perimeter ℓ hits a line.
 - (c) Assume that $\ell < 1$. Use part (a) and (b) to derive a formula for the probability that the needle hits a line. (**Hint:** The number of hits is a Bernoulli random variable.)