## Practice Midterm 1

1. 3 red balls and 3 blue balls are randomly arranged on a line. Let $X$ be the position of the first blue ball. (E.g. for the arrangement RBRBBR, $X=2$.) Find the probability mass function of $X$.
2. Half the students know the answer to a true-false question. The other half guesses at random. I ask a random student and his answer is correct. What is the probability he knows the answer?
3. Each pair of computers $a, b$ and $c$ is linked via a cable that fails with probability $10 \%$. Their failures are independent. Let $C_{x y}$ be the event $C_{x y}$ is the event "there is a working connection between computers $x$ and $y$." Are $C_{a b}$ and $C_{b c}$ independent?
4. The average lifetime of a lightbulb is 10 months. You install 10 lightbulbs on January 1. What is the probability that at least one of them fails in January? Assume their failures are independent.
5. Eight people's hats are mixed up and randomly redistributed. What is the expected number of pairs that exchanged hats (Alice got Bob's and Bob got Alice's)?

## Practice Midterm 2

1. Alice, Bob, Charlie, and Dave are randomly seated at a round table. The probability that Alice is seated next to Bob is $70 \%$. The probability that Bob is seated next to Charlie is $40 \%$. What is the probability that Charlie is seated next to Alice?
2. An unknown number of independent trials is performed, each of which succeeds with the same probability. You can only observe the number of successful trials. After many runs of this experiment you conclude that the expected number of successful trials is 6 , and the variance of this number is 2 . How many trials are performed?
3. Alice has a $\$ 1$, a $\$ 2$, and a $\$ 5$ coin. She randomly and secretly picks a coin with each hand (with equal probabilities) and shows the coin in her left hand to Bob. Bob may keep this coin or switch to the coin in Alice's right hand. Assuming Bob plays optimally what is his expected utility?
4. Trains reach Kowloon Station at an average rate of two trains per hour. Bob observed at least one train reach the station within the last hour. Given this information, what is the expected number of trains that reached the station within the last hour?
5. A dealer divides ten cards with face values $1,2, \ldots, 10$ among five players. Each player is randomly assigned two cards. A player wins if the sum of his cards' face values is 17 or higher. What is the expected number of winners? (Extra credit: What is the probability that there are no winners?)
