Practice questions

1. Coins A and B have probability of flipping heads equal to 0.01 and 0.1, respectively.
   (a) What is the expected number of heads in 100 tosses for the two coins?
   (b) For which one is there a higher chance of observing 5 heads in 100 tosses?
2. Let X and Y be the sum of 30 independent tosses of fair 3-sided dice with face values $\{0, 1, 2\}$ and $\{1, 2, 3\}$, respectively.
   (a) Calculate the means and variances of X and Y.
   (b) Apply the Central Limit Theorem to estimate the values $\mathrm{P}(X \geq 45)$ and $\mathrm{P}(Y \geq 45)$.
   (c) Use normal models for X and Y to estimate the ratio $\mathrm{P}(X = 45)/\mathrm{P}(Y = 45)$.
3. A mystery coin is either fair (type 1) or 70% heads, 30% tails (type 2). Your prior belief is that the coin is of type 1 with probablity 0.7 and type 2 with probability 0.3. Calculate the posterior probabilities after observing
   (a) a single head,
   (b) 7 heads in 10 flips.
4. Another mystery coin is fair (type 1), all heads (type 2), or all tails (type 3). Assume an equally likely prior on the three types. For any possible sequence of n outcomes derive a formula for the posterior probabilities.

Additional ESTR 2020 questions

5. Consider the following two methods for sampling a Binomial$(100, p)$ random variable Y. The method which was described in class outputs the smallest y for which $F(y) \geq U$, where F is the c.d.f. of Y and U is a Uniform$(0, 1)$ random variable. Another approach is to sample 100 independent Indicator(p) random variables and output their sum. Which method uses fewer random bits in expectation? Which one is faster? What changes if you have to output multiple independent samples?
6. Try to prove inequality $(\star\star)$ from the ESTR lecture 1 notes.