Practice questions

1. The PDF of a random variable is either $f_0(x) = 1/2$ (H_0) or $f_1(x) = 1/(\pi \cdot \sqrt{1-x^2})$ (H_1), where $-1 < x < 1$.

   (a) For a given threshold $t > 0$, describe the set of values x for which $f_1(x)/f_0(x) \geq t$.

   (b) Use part (a) and the Neyman-Pearson lemma to design a test (for a single sample) with false positive probability 1/4.

   (c) What is the false negative probability of your test?

2. You are given ten samples of a Uniform$(0, \theta)$ random variable. You want to test whether $\theta \geq 1$ (H_0) or $0 \leq \theta < 1$ (H_1). Consider the test that accepts if all ten samples have value less than 3/4.

   (a) What is the largest possible false positive probability of this test?

   (b) Calculate the power function of this test, i.e., the probability that the test accepts for a given $\theta \in H_1$.

   [Adapted from DS textbook problem 9.1.2]

3. The reported daily traffic of an amusement park is 11,000 people. Your alternative hypothesis is that it should be at least 12,000 people.

   (a) Your observation in one day is a Normal$(\mu, 500)$ random variable where μ is the true daily traffic. You observed 11,800 people in a particular day. What is the p-value for your hypothesis?

   (b) How many (independent) days of observation do you need to test your hypothesis with a 10% false positive and a 10% false negative probability?

4. You suspect that when humans type long "random" strings (sequences of `0`s and `1`s) they tend to avoid long consecutive blocks with the same value. To test your hypothesis you design the following experiment: Ask each of 100 subjects to write a random 10-bit string. Then count the number X of answers in which all four middle bits are identical (`0000` or `1111`).

   (a) If the answers were truly random, what kind of random variable would X be?

   (b) State the null hypothesis (based on part (a)) and your alternative hypothesis.

   (c) Design a test for your hypothesis with a 10% false positive error.