1. Alice obtained a score of 26 on her ENGG 2780 exam. Her five friends' scores are 29, 27, 19, 24, and 17. Assume the scores are independent samples of a normal random variable.

   (a) Can Alice conclude that she did better than average with 75% confidence?

   (b) What is the p-value for the hypothesis in part (a)?

2. CUHK and HKU students took part in a math exam and obtained the following scores:

team	number of students	mean score	standard deviation
CUHK	80	84	12.4
HKU	100	80	11.2

   Assuming the students were chosen at random calculate the p-value for the hypothesis "CUHK does better than HKU in math" if the listed standard deviations are (a) the actual ones and (b) the adjusted sample deviations.

3. A cookie manufacturer wants to test if replacing milk chocolate with dark chocolate in their product will lower the calorie count. To do so it creates sixteen cookie batches, tests the samples, and obtains the following numbers:

with dark chocolate	113	120	138	120	100	118	138	123
with milk chocolate	138	116	125	136	110	132	130	110

   (a) If the batches were produced by eight (independent) cooks, each of which made one with dark and one with milk chocolate, which test would be appropriate to use, and what is the p-value?

   (b) If one cook produced all the dark chocolate batches and another one produced all the milk chocolate ones, which test would you use? How does the p-value change?

4. You are given two samples X_1, X_2 of a Uniform$(0, \theta_1)$ random variable and two samples Y_1, Y_2 of a Uniform$(0, \theta_2)$ random variable, all independent. You want to test the alternative hypothesis $\theta_2 > \theta_1$ against the null hypothesis $\theta_2 = \theta_1$. Consider the test T that accepts if $\min\{Y_1, Y_2\} > \max\{X_1, X_2\}$ and rejects if not.

   (a) What is the false positive error of T?

   (b) What is the power (acceptance probability) of T as a function of $\rho = \theta_2/\theta_1$?

   (c) **(Optional)** What is the likelihood ratio test, i.e., the test of the form

$$\frac{\sup_{H_1} f(x_1, x_2, y_1, y_2|\theta_1, \theta_2)}{\sup_{H_0} f(x_1, x_2, y_1, y_2|\theta_1, \theta_2)} \geq t$$

   (where f is the joint PDF of the four samples) for a 1/4 false positive error?