Practice questions

1. The Department of Transportation reports the following numbers of accidents in different days of the week:

M	T	W	T	F	S	S
35	23	29	31	34	60	25

   (a) You suspect that the chance of an accident depends on the day of the week. State the null hypothesis and calculate the p-value for your (alternative) hypothesis.

   (b) You suspect that the chance of an accident is different on weekdays and weekends. What is the p-value now?

2. You observe the following sorted sequence of samples of a continuous random variable, which is hypothesized by default to be Exponential(1).

0.013	0.018	0.066	0.086	0.136	0.138	0.172
0.311	0.321	0.654	0.828	1.060	1.326	1.373
1.682	1.860	2.232	3.191	3.715	3.720	5.780

   (a) How should you partition the range of an Exponential(1) random variable X into three intervals I_1, I_2, I_3 so that $\mathrm{P}(X \in I_1) = \mathrm{P}(X \in I_2) = \mathrm{P}(X \in I_3) = 1/3$?

   (b) What is the p-value for the chi-square test with respect to the partition in part (a)? Does it support an alternative hypothesis?

3. A hospital is performing an experiment about the effect of different methods in administering a drug. Apply the chi-square test for independence to determine the p-value for the null hypothesis that the effect is independent of the administration method.

	Effective	Ineffective	Number
Oral	58	40	98
Injection	64	31	95
Sum	122	71	193

4. In this question you will prove the correctness of the chi-square test for discrete random variables that take two values.

   (a) Show that the chi-square statistic X^2 for n samples of an Indicator(p) random variable, N of which come up positive, has value $(N - np)^2/(np(1-p))$.

   (b) Show that $X^2 \geq t^2$ if and only if $|N - \mu| \geq t\sigma$, where μ and σ are the Binomial(n, p) mean and standard deviation, respectively.

   (c) Using the central limit theorem, show that under the null hypothesis, as n gets large, $\mathrm{P}(X^2 \geq t^2)$ approaches $\mathrm{P}(Y \geq t^2)$, where Y is a $\chi^2(1)$ random variable.