Each question is worth 10 points. Explain your answers clearly.

1. You observe a single sample X of a Uniform$(-\Theta, \Theta)$ random variable with Bayesian Uniform$(0, 1)$ prior on the parameter Θ.
   (a) If $X = 0.5$, what is the posterior PDF of Θ?
   (b) What is the conditional expectation of Θ given $X = 0.5$?
2. You observe one sample of a random variable that is either Uniform$(-2, 2)$ (null hypothesis) or Normal$(0, 1)$ (alternative hypothesis).
   (a) Describe the test with the smallest false negative error that achieves false positive error $1/4$.
   (b) What is the false negative error of your test from part (a)?
3. The number of complaints that TV station X receives from its viewers in a given day is modeled by a Normal$(\mu, 10)$ random variable for some unknown μ that models viewer unhappiness. Station X received $M = 95$ complaints on Monday and $T = 130$ complaints on Tuesday.
   (a) Assuming M and T are independent, what kind of random variable is $T - M$?
   (b) What is the p-value for the null hypothesis that unhappiness didn't increase?
4. You observe the following statistics in 100 tosses of a possibly unfair 3-sided die:

outcome:	1	2	3
count:	27	54	19

   You want to test null hypothesis that outcomes 1 and 3 each have probability θ and outcome 2 has probability $1 - 2\theta$ for some unknown θ.
   (a) What is the maximum likelihood estimate of θ from your data?
   (b) Can you reject the null hypothesis at the 95% confidence level?

Each question is worth 10 points. Explain your answers clearly.

1. You are observing a single sample of a random variable which is either Uniform$(0, 2)$ $(\Theta = 0)$ or Uniform$(1, 5)$ $(\Theta = 1)$. Your prior on Θ is $\mathrm{P}(\Theta = 0) = 1/5$ and $\mathrm{P}(\Theta = 1) = 4/5$.
   (a) What is the MAP estimator of Θ?
   (b) What is the error probability $\mathrm{P}(MAP \neq \Theta)$?

2. Let X_1, X_2 be two independent samples of a Uniform$(0, 1)$ random variable.
   (a) Calculate the PDF of the sample mean.
   (b) What is the probability that the actual mean is within $1/4$ of the sample mean?

3. Out of 100 customers polled (randomly with repetition) about the popularity of a new toothpaste, 60 were positive.
   (a) What is the sample mean and sample standard deviation of the popularity of the product?
   (b) Give a 90% confidence interval for the true popularity. Explain which formula you are using.

4. A review of typists' work at a publishing company on a given day yielded the following data:

	Alice	Bob	Charlie
morning shift typos	18	16	10
afternoon shift typos	22	14	12

   You want to test the null hypothesis that the number of typos doesn't change between shifts. Assume that each data item comes from normal random variables with the same variance, and that the performance of different typists is independent.
   (a) Would you use the independent sample test or the paired t-test? Justify your answer.
   (b) Calculate the p-value for the test you chose in part (a).

Each question is worth 10 points. Explain your answers clearly.

1. A hypothesis test T has a false positive (type I) error of 10%. Assume that the null hypothesis is true.
   (a) You run the test twice on independent data. What is the probability that the test produces inconsistent results?
   (b) You run the test one more time. What is the probability that a majority (at least 2 out of 3) of test results reject the null hypothesis?
2. A random sample yields the following E.Coli bacteria counts in four one-litre samples of water from a river: 229, 354, 356, 344. Assume that the population counts are independent normal with the same mean and variance.
   (a) Find the sample mean and the adjusted sample variance.
   (b) Construct a 95% confidence interval for the true mean of the population.
   (c) Give 95% upper confidence bound for the true variance of the population.
3. A coin is tossed 252 times.
   (a) Let h be the fraction of heads observed. Which values of h would indicate that the coin is biased (either towards heads or tails) at the 90% confidence level?
   (b) The coin comes up heads 133 times. Can you conclude that it is biased towards tails at the 90% confidence level?
4. A normal random variable of mean zero has standard deviation either $\sigma = 1$ (null hypothesis) or $\sigma = 2$ (alternative hypothesis).
   (a) Given one sample of this random variable, describe the test with false positive (type I) error of 20% that has the smallest false negative (type II) error.
   (b) What is the false negative (type II) error of your test?