Alice is about to toss one of two 3-sided dice. The first die is fair with face values `1`, `2`, `3`. The second die has face value `1` on all sides. Bob's prior probabilities are 90% on the first die and 10% on the second die. Alice tosses the die and the toss is a `1`. Calculate Bob's posterior probabilities.

Solution: Let Θ and X denote the die and the outcome, respectively. By Bayes' rule,

$$\mathrm{P}(\Theta = 1|X = 1) \propto \mathrm{P}(X = 1|\Theta = 1) \cdot \mathrm{P}(\Theta = 1) = \frac{1}{3} \cdot 0.9 = 0.3$$
$$\mathrm{P}(\Theta = 2|X = 1) \propto \mathrm{P}(X = 1|\Theta = 2) \cdot \mathrm{P}(\Theta = 2) = 1 \cdot 0.1 = 0.1$$

As the posterior probabilities must add up to one, we get

$$\mathrm{P}(\Theta = 1|X = 1) = \frac{0.3}{0.3 + 0.1} = \frac{3}{4} \quad \text{and} \quad \mathrm{P}(\Theta = 2|X = 1) = \frac{0.1}{0.3 + 0.1} = \frac{1}{4}.$$