The lap time of a runner at the CUHK track is a normal random variable of mean μ and standard deviation 10 (in seconds), where μ is 60 for an athlete and 120 for an amateur runner. Only 1% of the runners are athletes. Bob runs a lap in 80 seconds. How does the MAP rule classify Bob?

Solution: Let Θ be type of runner (0 for amateur, 1 for athlete) and X be the runner's lap time. The prior is $\mathrm{P}(\Theta = 0) = 0.99$, $\mathrm{P}(\Theta = 1) = 0.01$ and the likelihood is $f_{X|\Theta}(x|\theta) = (2\pi 10^2)^{-1/2} e^{-(x-\mu(\theta))^2/2\cdot 10^2}$, where $\mu(0) = 120$ and $\mu(1) = 60$. The ratio of posteriors is

$$\frac{\mathrm{P}(\Theta = 1|X = 80)}{\mathrm{P}(\Theta = 0|X = 80)} = \frac{\mathrm{P}(\Theta = 1) \cdot f_{X|\Theta}(80|1)}{\mathrm{P}(\Theta = 0) \cdot f_{X|\Theta}(80|0)} = \frac{0.01 \cdot (2\pi 10^2)^{-1/2} e^{-(80-60)^2/2\cdot 10^2}}{0.99 \cdot (2\pi 10^2)^{-1/2} e^{-(80-120)^2/2\cdot 10^2}} = \frac{e^6}{99}.$$

As $e^6 \approx 403$ the ratio exceeds one so the MAP rule classifies Bob as an athlete.