What is the PMF of the sample minimum of two tosses of a fair 3-sided die with outcomes 1, 2, 3?

Solution: Let X_1 and X_2 be the two outcomes and M their minimum. By independence, for $i \in \{1, 2, 3\}$

$$
\begin{aligned}
P(M \geq i) &= P(X_1 \geq i \text{ and } X_2 \geq i) \\
&= P(X_1 \geq i) \, P(X_2 \geq i) \\
&= (1 - P(X_1 < i))(1 - P(X_2 < i)) \\
&= \left(1 - \frac{i-1}{3}\right)^2.
\end{aligned}
$$

Therefore $P(M \geq 2) = 4/9$ and $P(M \geq 3) = 1/9$. The PMF is

$$
\begin{array}{c|ccc}
m & 1 & 2 & 3 \\
P(M = m) & \frac{5}{9} & \frac{1}{3} & \frac{1}{9}
\end{array}
$$